Capacity and Coded Performance Of Optical Binary DPSK

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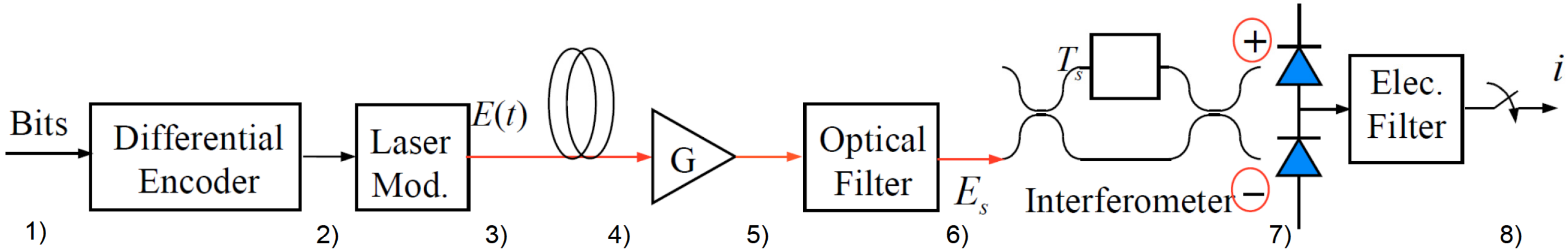
Optical DPSK: The Basics

Data flow of optical binary differential phase-shift keying (DPSK):



1) Coded bits enter a differential encoder
2) Differentially encoded bits modulate the phase of a laser pulse:
   - 0: transmit a pulse with phase 0
   - 1: transmit a pulse with phase π
3) Laser light is transmitted into free space
4) Light enters telescope (not shown), and is focused on an optical amplifier, which:
   - Applies a gain G to the optical field
   - Introduces Amplified Spontaneous-Emission (ASE) noise
     - This is modeled using Gaussian statistics
     - In the ideal amplifier, the noise variance is equal to the squared signal power
     - This is the dominant source of noise in the system, which means that performance can be stated in terms in terms of signal entering amplifier, without any reference to noise or SNR.
5) Light enters optical filter, which eliminates noise not at the frequency or polarization of the signal
6) Light enters a delay-line interferometer (DLI), which acts as a differential detector:
   - Light is split into two equal-intensity components
   - One component is delayed by one symbol period
   - The two fields enter an interferometer, which produces outputs at “sum” and “difference” ports
7) The two port outputs enter photodetectors, and a difference of the two outputs is formed
8) The soft demodulator output

Mathematical Model of DPSK

- When there is no transition from one symbol to the next, the probability density functions (pdfs) at the output of the sum port and the difference port are:

$$p_S(x) = 2\, e^{-2(x+N_p)} I_0\left(4\sqrt{xN_p}\right)$$

$$p_N(x) = 2\, e^{-2x}$$

where

- N_p is the number of photons per channel use at the input to the optical amplifier
- $I_0(x) = \frac{1}{\pi}\int_0^{\infty} \exp(x cos\theta)\, d\theta$ is the modified Bessel function of the first kind of order 0.

- When there is a transition, the two pdfs are switched. Thus, $S - N$ is a sufficient statistic.
- The difference of the two ports has the pdf:

$$p_D(d) = \int_0^{\infty} p_S(x) p_N(x-d) dx = e^{2d-N_p}\, Q\left(\sqrt{2N_p}, \sqrt{8\max(0,d)}\right) \quad \text{(many steps are skipped)}$$

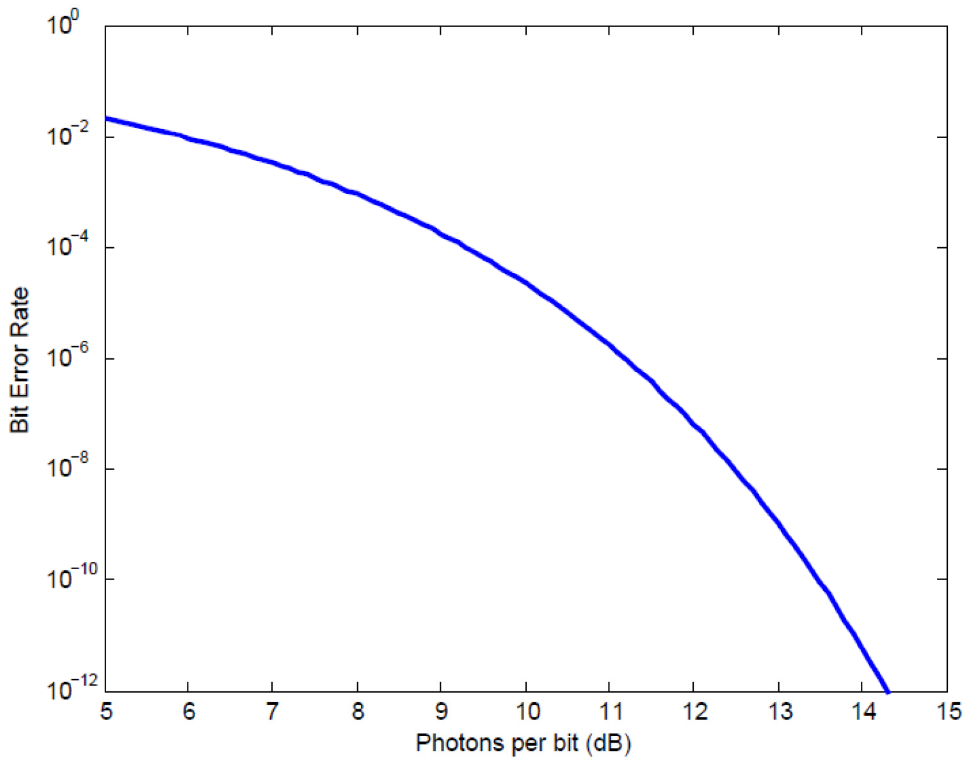
where $Q(a,b) = \int_b^{\infty} x \exp\left(-\frac{x^2+a^2}{2}\right) I_0(\,ax) dx$ is the Marcum Q function of order 1.

BER of Uncoded DPSK

The bit error rate (BER) of DPSK is given by the probability the demodulator output is negative, given that there was no bit transition (it should be positive):

$$P_e = \int_{-\infty}^{0} p_D(x)dX = \frac{1}{2}e^{-N_p}$$

(many simplifying steps are skipped)



Capacity of DPSK

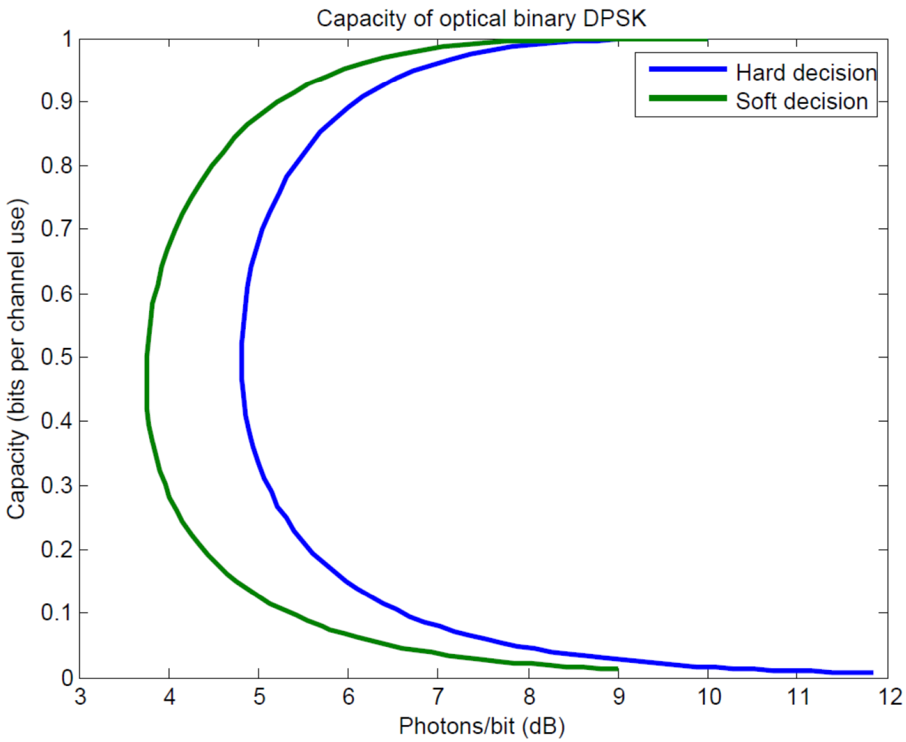
When hard decisions are made on demodulator output, capacity is given by:

$$C = 1 + P_e \log_2 P_e + (1 - P_e) \log_2 (1 - P_e)$$

When soft decisions are made, capacity is given by:

$$C = 1 + \int_{-\infty}^{\infty} p_D(x) \log_2 \left[\frac{p_D(x)}{p_D(x) + p_D(-x)} \right] dx$$

With careful numerical evaluation, this gives plot shown here



Notes:

- Results are expressed vs. photons/bit (N_p/C), not photons per channel use (N_p).
- The optimal rate is very close to ½.
- This is unlike the RF channel, where lower rates are always more efficient.

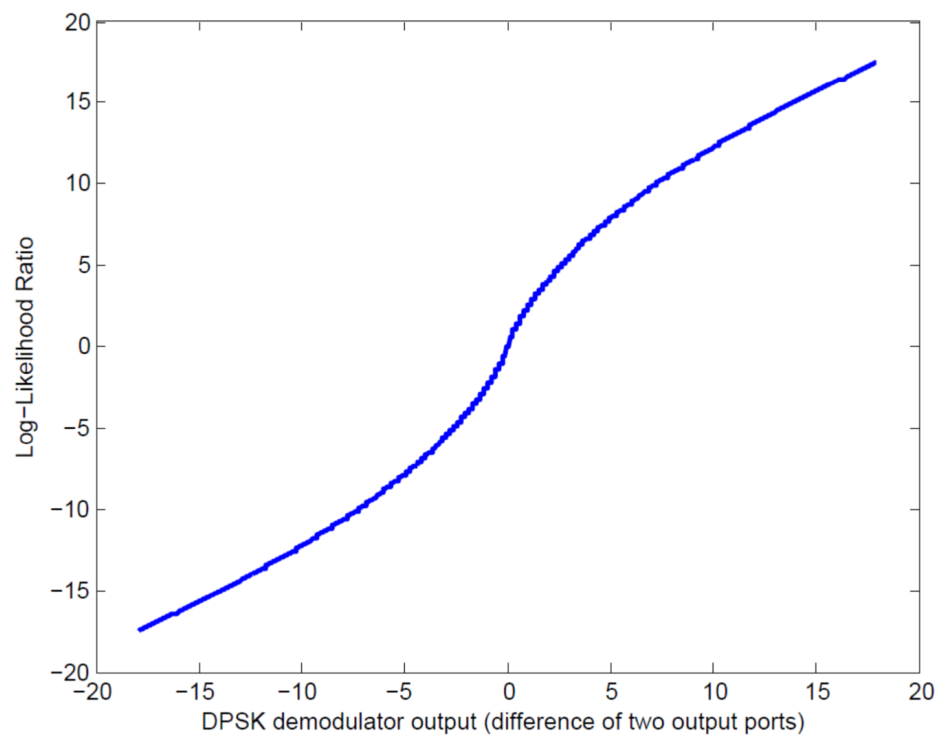
Converting Demodulator Output to Decoder Input

Modern channel codes use soft decisions, in the form of a log-likelihood ratio:

$$\lambda(d) = \ln\left[\frac{p_D(d|b=0)}{p_D(d|b=1)}\right] = 4d + \mathrm{sgn}(d)\ln\left[\frac{Q\left(\sqrt{2N_p}, \sqrt{8|d|}\right)}{Q\left(\sqrt{2N_p}, 0\right)}\right]$$

(many simplifying steps are skipped)

LLR for $N_p = 2$:



Notes:

- Near 0 there is a linear region with slope 4.
- Special care should be taken to ensure robust calculation of the Marcum Q function.

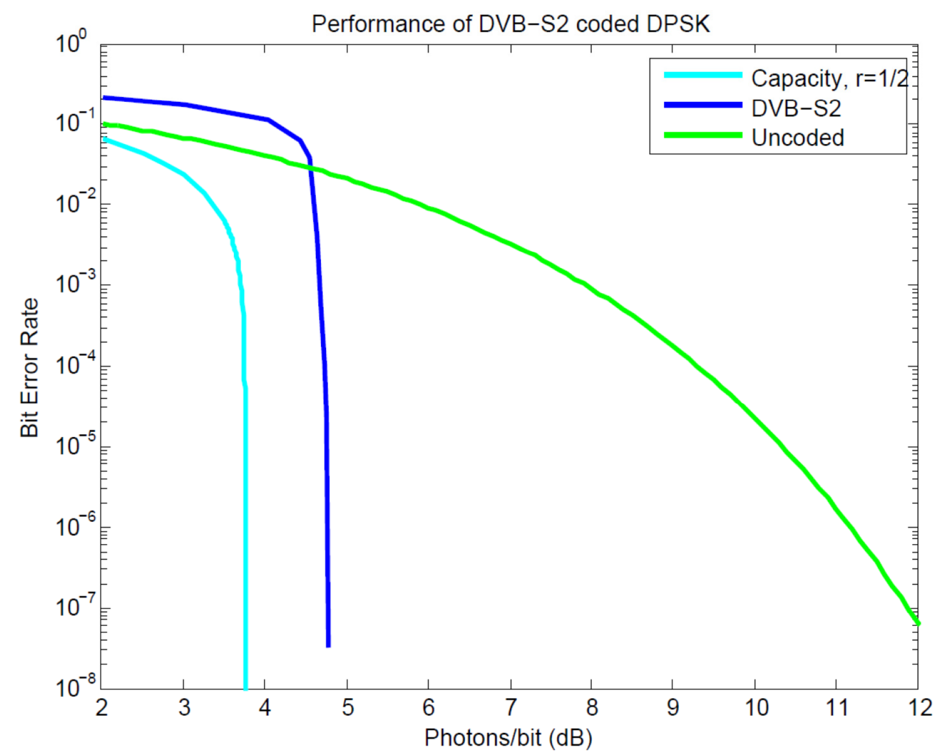
The LLR is the input to the decoder.

Coded Performance of DPSK

- Optical DPSK behaves in many ways like a regular AWGN or fading RF channel
- Thus, codes that are good for the RF channel are generally good for optical DPSK
- Emphasis should be placed on codes near rate ½, since that is optimal for optical DPSK.

- Shown at the right:
    - Capacity of rate ½ coded DPSK
    - DVB-S2 (n=64800,r=1/2) coded DPSK
    - Uncoded DPSK

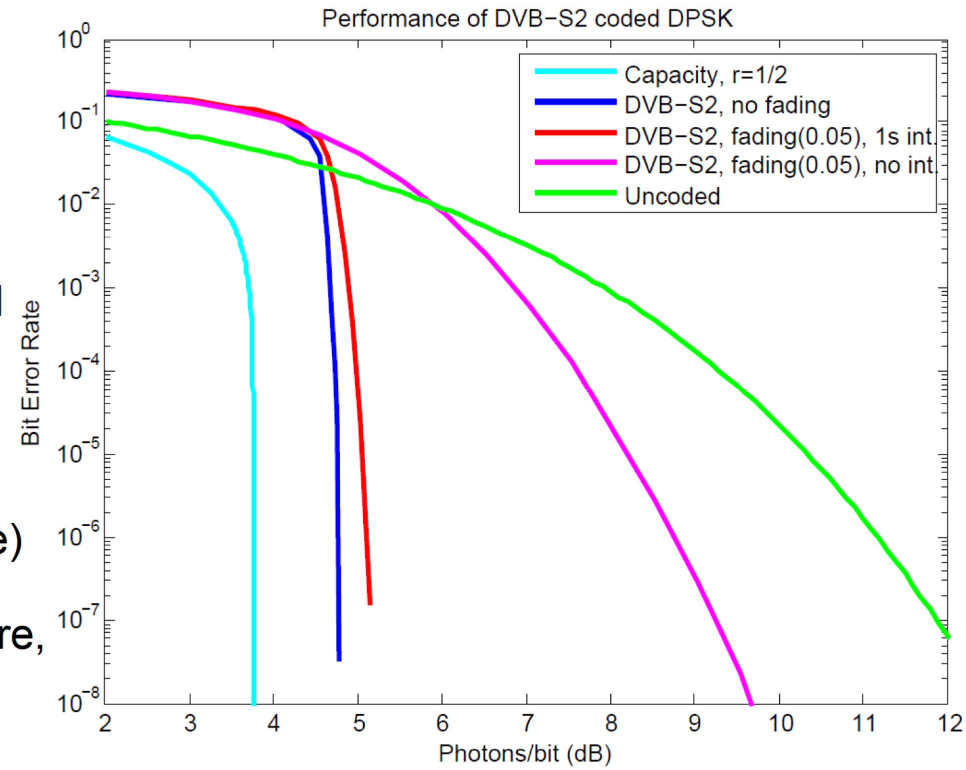
- Performance with any other good code, such as AR4JA LDPC, would be similar



Coded Performance in Fading

- Free space optical channel is a fading channel, because of the atmosphere
- It is well-modeled as lognormal fading

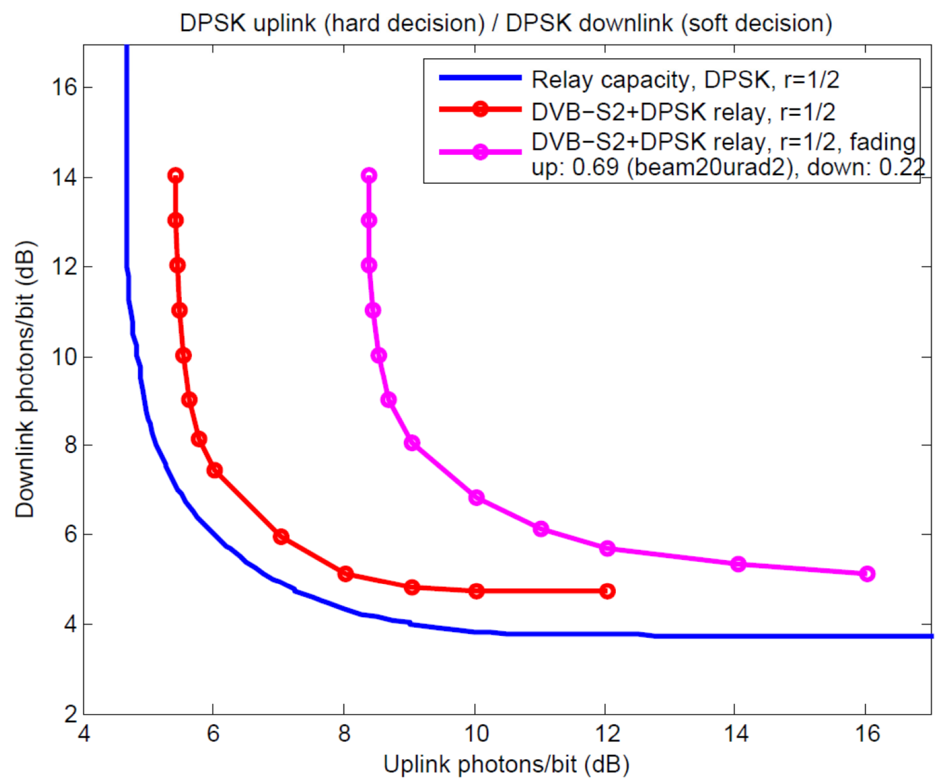
- Case study:
    - Data rate: 1.244 Gbps
    - Lognormal fading with unit mean and $\sigma^2 = 0.05$
    - Coherence time of 7 ms
    - Block fading (independent fades applied on a block basis, with blocklengths equal to coherence time)
- Performance for case study shown at right
    - Capacity, DVB-S2, uncoded, as before, with no fading
    - DVB-S2 with fading, no channel interleaver used
    - DVB-S2 with fading, 1 s channel interleaver applied



- Channel interleaver is critical to combat fading

Relay Performance

- Relay scenario:
  - DVB-S2 coded DPSK for uplink
  - Hard demodulator on spacecraft
  - No decoding on spacecraft
  - Remodulate with DPSK for downlink
  - Soft decision decode on ground
- Representative of broadcast application



Shown at right is the uplink photons/bit and downlink photons/bit needed to close link

- Capacity of DPSK, rate ½
- DVB-S2 coded DPSK
- DVB-S2 coded DPSK with 1s interleaver, on fading channel with $\sigma = 0.69$ on the uplink and $\sigma = 0.22$ on the downlink.

Notes:

- >4.7 photons/bit (dB) are needed on uplink
- >3.7 photons/bit (dB) are needed on downlink

References

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Conclusions

- The ideal optical DPSK channel is fairly well understood
  - Statistics are well-modeled as Gaussian
  - Uncoded performance is same as RF DPSK channel in AWGN
  - Capacity is known
- Optical DPSK behaves in many ways like an AWGN channel
- One important difference: rate ½ is optimal
- Good codes for AWGN will be good for optical DPSK
- Standard channel interleaving techniques do good job of combating fading